The University of Texas at Austin
Electrical and Computer Engineering
Cockrell School of Engineering

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INTRODUCTION TO COMPUTER VISION

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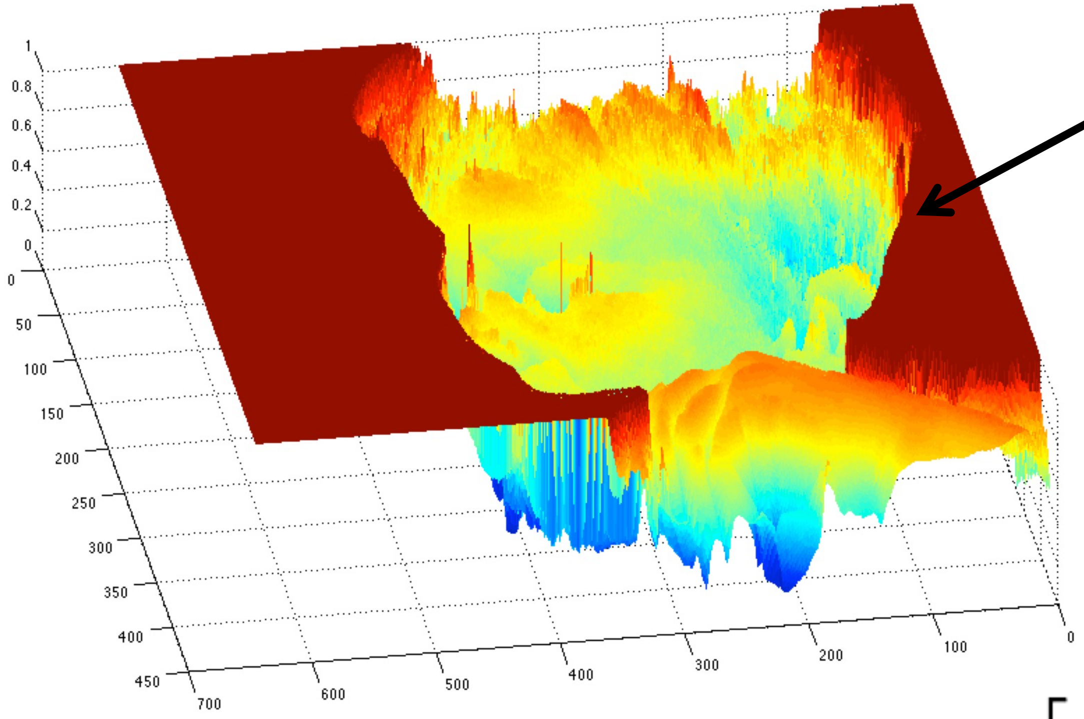
What are image edges?

grayscale image

$f(\boldsymbol{x})$



Very sharp discontinuities in intensity.

domain $\boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

Detecting edges

How would you go about detecting edges in an image (i.e., discontinuities in a function)?

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✓ You take derivatives: derivatives are large at discontinuities.

How do you differentiate a discrete image (or any other discrete signal)?

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How would you go about detecting edges in an image (i.e., discontinuities in a function)?

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How do you differentiate a discrete image (or any other discrete signal)?

✓ You use finite differences.

Finite differences

High-school reminder: definition of a derivative using forward difference

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Finite differences

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Alternative: use central difference

$$f'(x) = \lim_{h \to 0} \frac{f(x+0.5h) - f(x-0.5h)}{h}$$

For discrete signals: Remove limit and set h = 2

$$f'(x) = \frac{f(x+1) - f(x-1)}{2}$$

What convolution kernel does this correspond to?

Finite differences

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For discrete signals: Remove limit and set h = 2

$$f'(x) = \frac{f(x+1) - f(x-1)}{2}$$

1D derivative filter

1	0	-1

The Sobel filter

Vertical Sober filter:

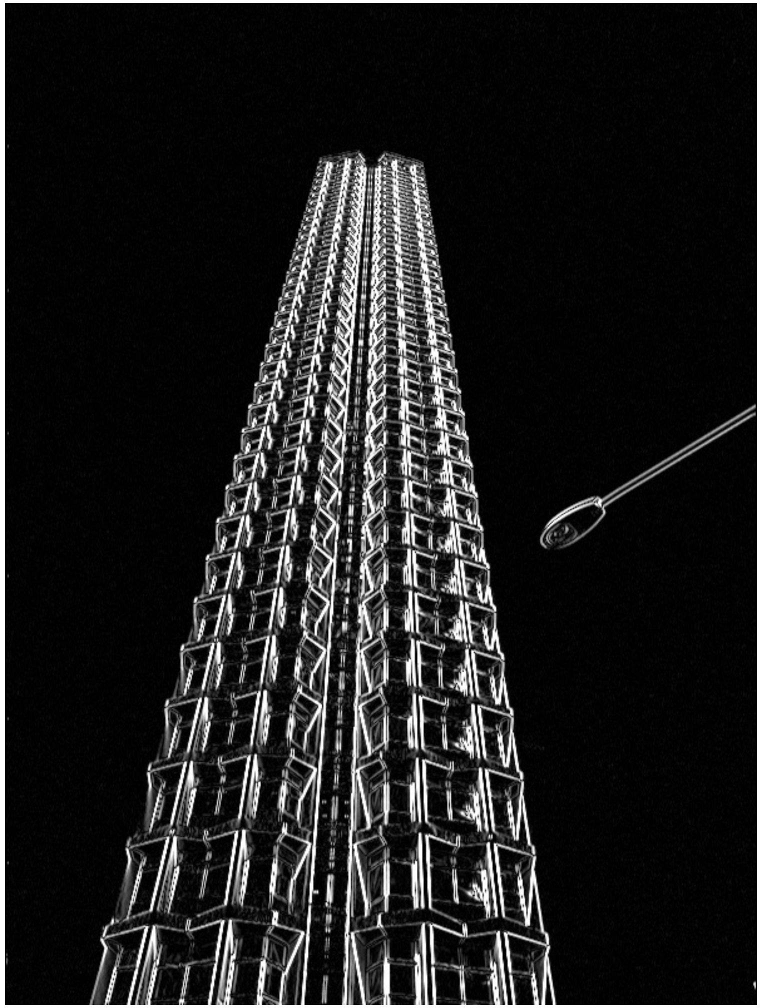
$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}_{\text{“Blurring”}} * \underbrace{\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}}_{\text{“Derivative”}}$$

Horizontal Sobel filter:

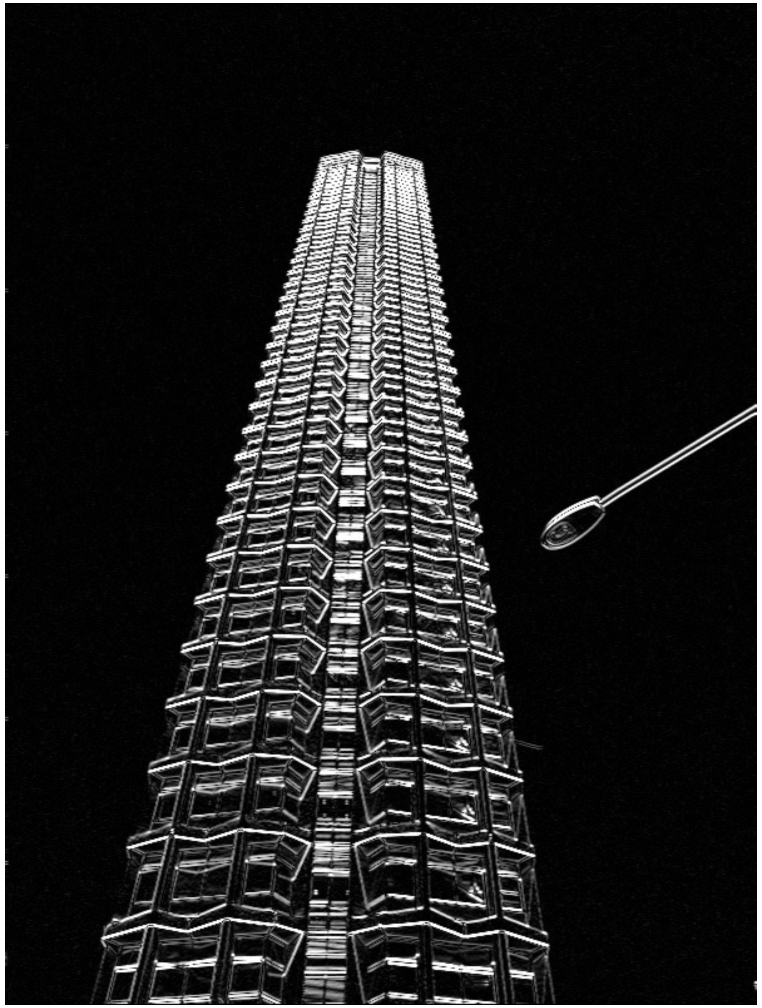
$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}_{\text{“Derivative”}} * \underbrace{\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}}_{\text{“Blurring”}}$$

Sobel filter example

original



which Sobel filter?



which Sobel filter?

Computing image gradients

1. Select your favorite derivative filters.

$$\boldsymbol{S}_x = \begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} \qquad \boldsymbol{S}_y = \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

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2. Convolve with the image to compute derivatives.

$$\frac{\partial \boldsymbol{f}}{\partial x} = \boldsymbol{S}_x \otimes \boldsymbol{f} \qquad \frac{\partial \boldsymbol{f}}{\partial y} = \boldsymbol{S}_y \otimes \boldsymbol{f}$$

Computing image gradients

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2. Convolve with the image to compute derivatives.

$$\frac{\partial \boldsymbol{f}}{\partial x} = \boldsymbol{S}_x \otimes \boldsymbol{f} \qquad \frac{\partial \boldsymbol{f}}{\partial y} = \boldsymbol{S}_y \otimes \boldsymbol{f}$$

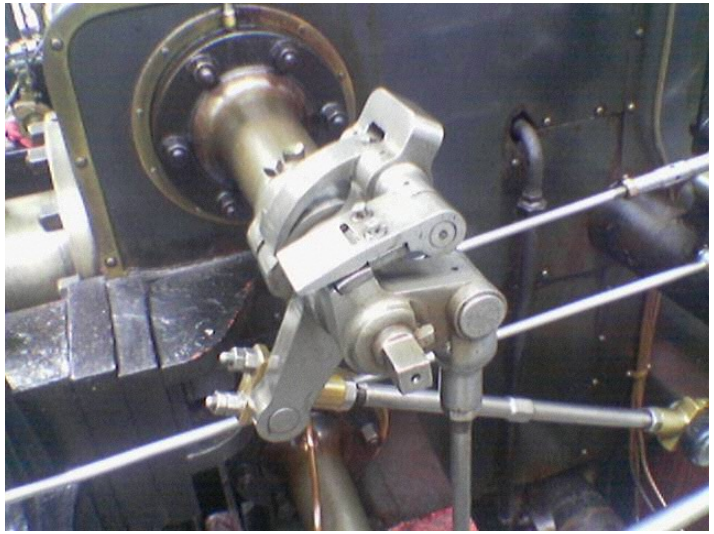
3. Form the image gradient, and compute its direction and amplitude.

$$\nabla \boldsymbol{f} = \left[\frac{\partial \boldsymbol{f}}{\partial x}, \frac{\partial \boldsymbol{f}}{\partial y}\right] \qquad \theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right) \qquad ||\nabla f|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

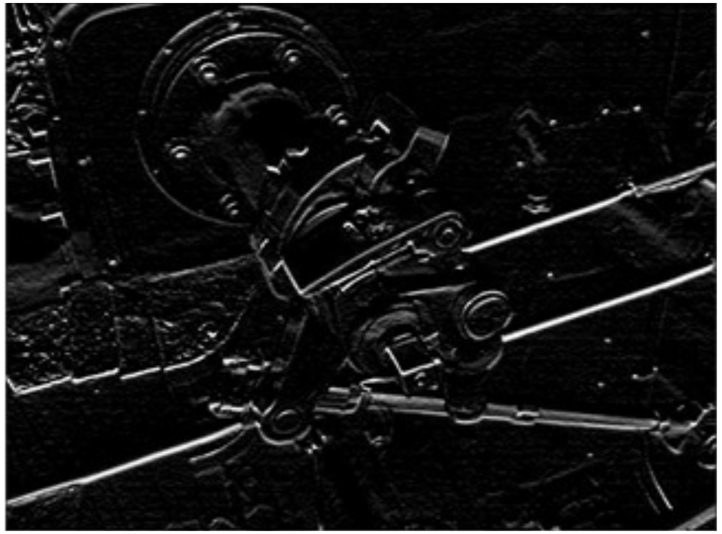
gradient direction amplitude

Image gradient example

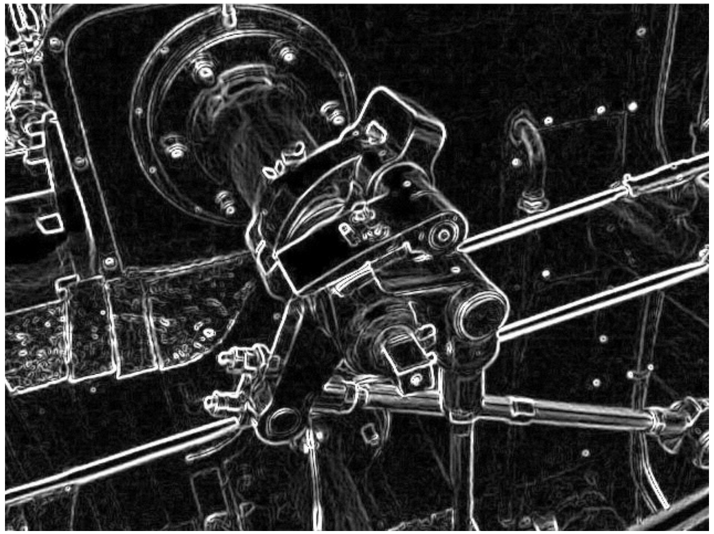
original



vertical
derivative



gradient
amplitude

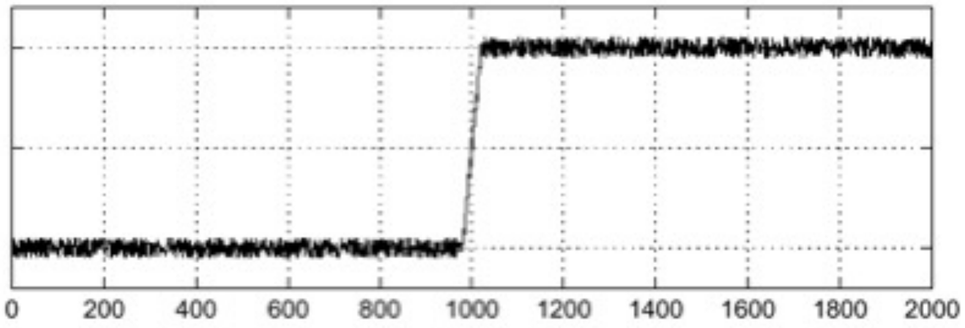


horizontal
derivative

How does the gradient direction relate to these edges?

How do you find the edge of this signal?

intensity plot



How do you find the edge of this signal?

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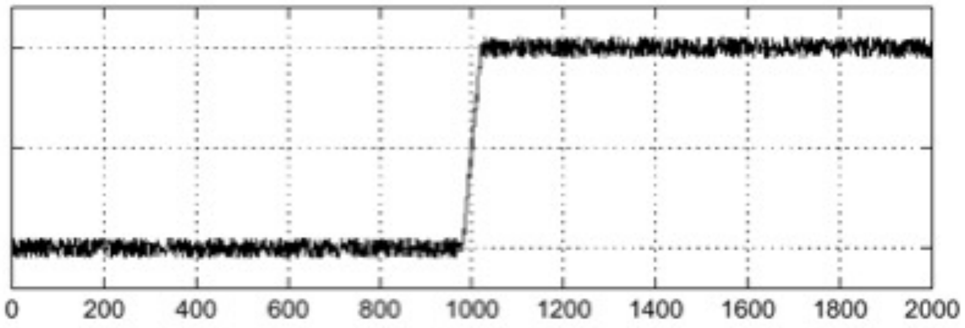
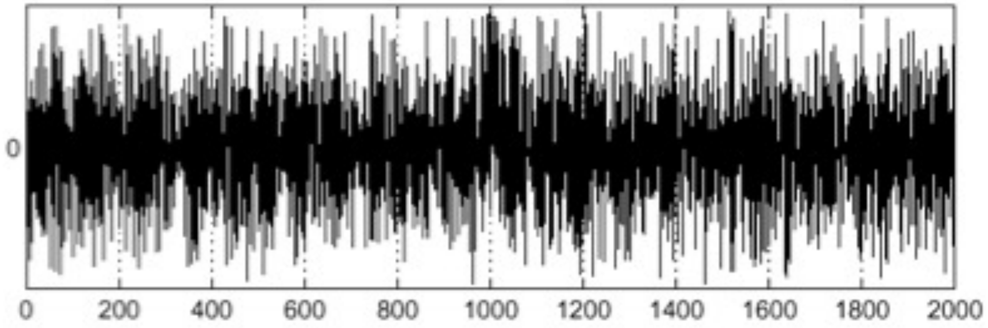


Using a derivative filter:

derivative plot



What's the problem here?

Differentiation is very sensitive to noise

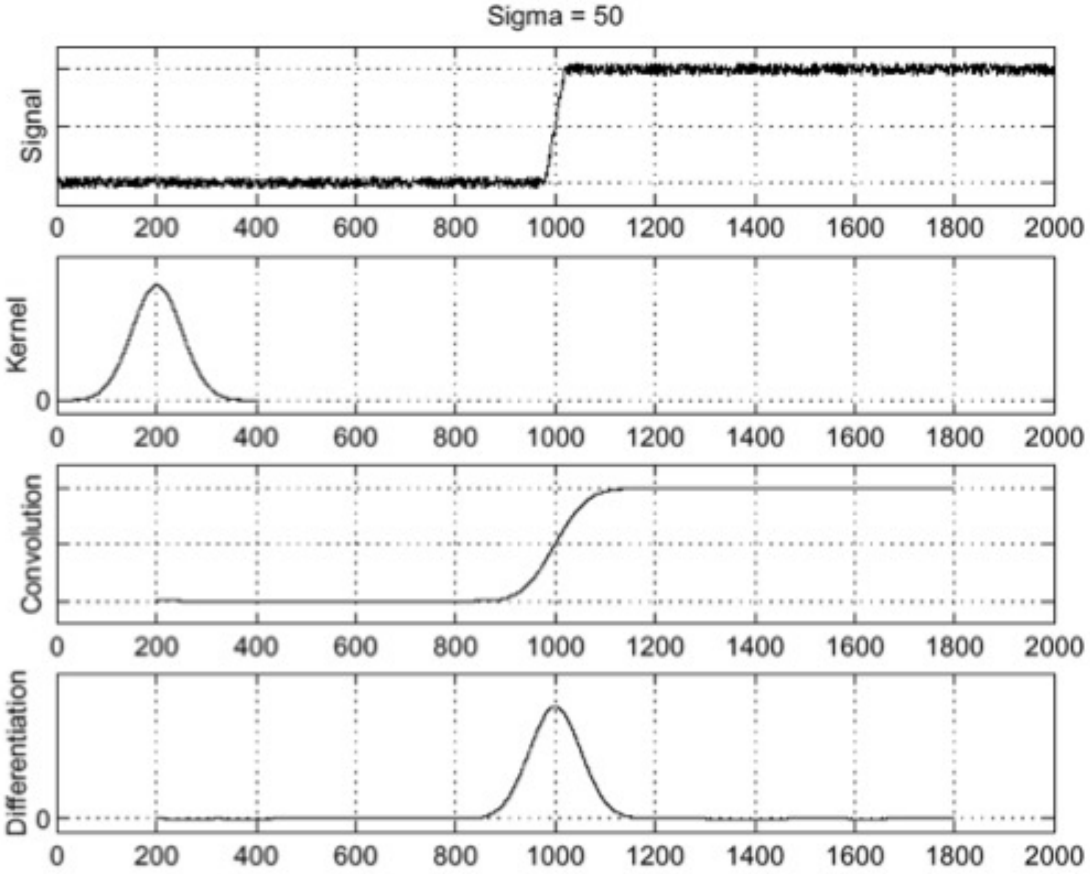
When using derivative filters, it is critical to blur first!

input

Gaussian

blurred

derivative of
blurred



How much
should we blur?

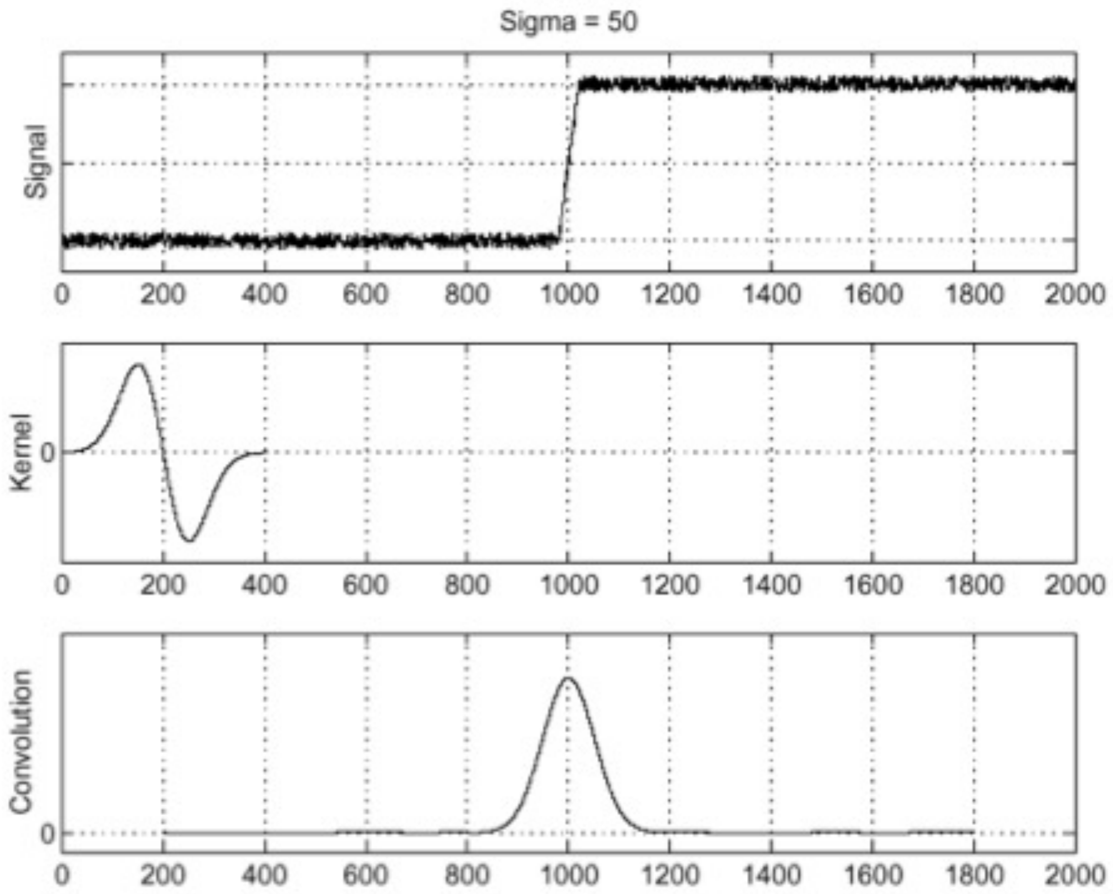
Derivative of Gaussian (DoG) filter

Derivative theorem of convolution: $\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$

input

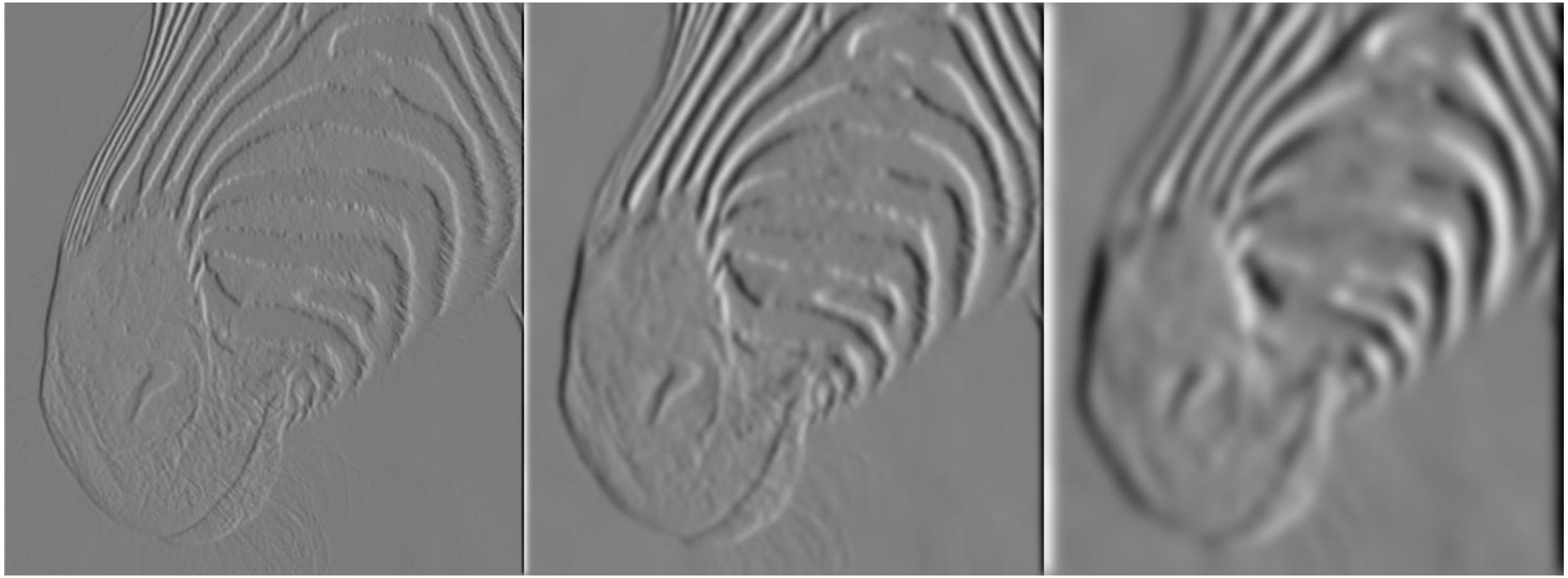
derivative of Gaussian

output (same as before)



- How many operations did we save?

Tradeoff between smoothing and localization



1 pixel 3 pixels 7 pixels

- Smoothed derivative removes noise, but blurs edge. Also finds edges at different "scales".

Source: D. Forsyth

Laplace filter

Basically a second derivative filter.

- We can use finite differences to derive it, as with first derivative filter.

first-order finite difference

$$f'(x) = \lim_{h \to 0} \frac{f(x+0.5h) - f(x-0.5h)}{h} \longrightarrow$$

1D derivative filter

1	0	-1

second-order finite difference

$$f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \longrightarrow$$

Laplace filter

?

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$$f''(x) = \lim_{h \to 0} \frac{f(x + h) - 2f(x) + f(x - h)}{h^2} \longrightarrow$$

Laplace filter

1	-2	1

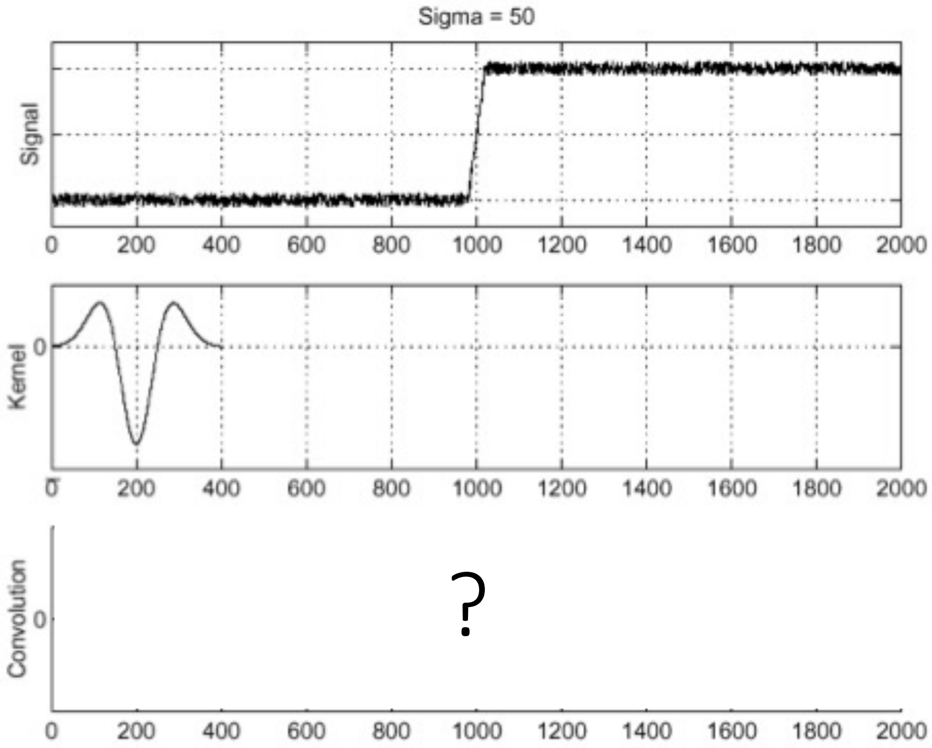
Laplacian of Gaussian (LoG) filter

As with derivative, we can combine Laplace filtering with Gaussian filtering

input

Laplacian of Gaussian

output



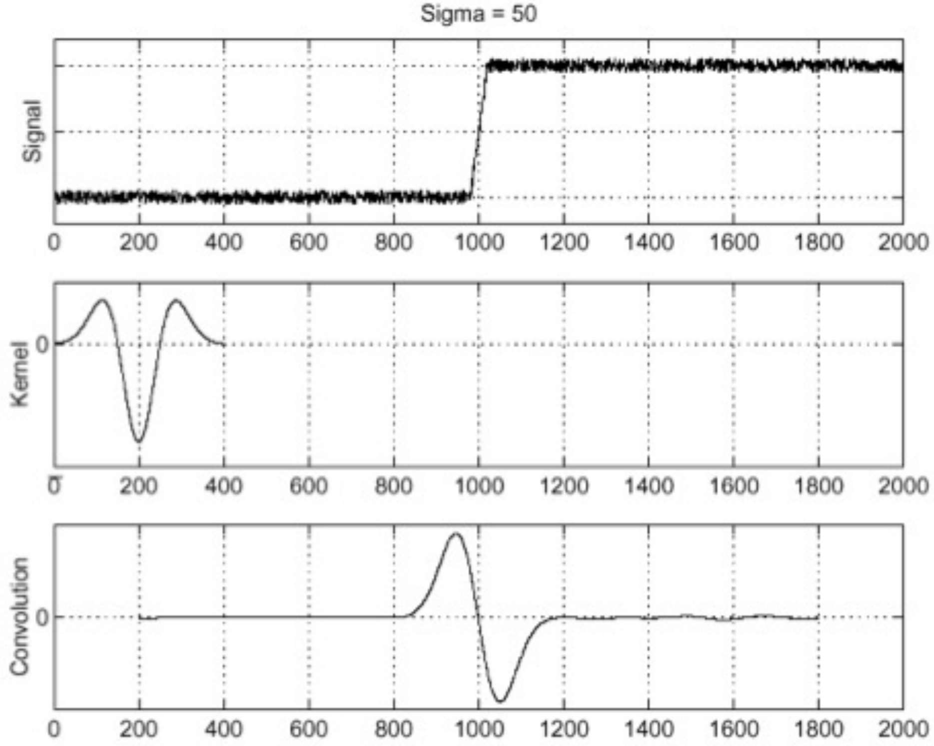
Laplacian of Gaussian (LoG) filter

As with derivative, we can combine Laplace filtering with Gaussian filtering

input

Laplacian of Gaussian

output



"zero crossings" at edges

Laplace and LoG filtering examples

Laplacian of Gaussian filtering

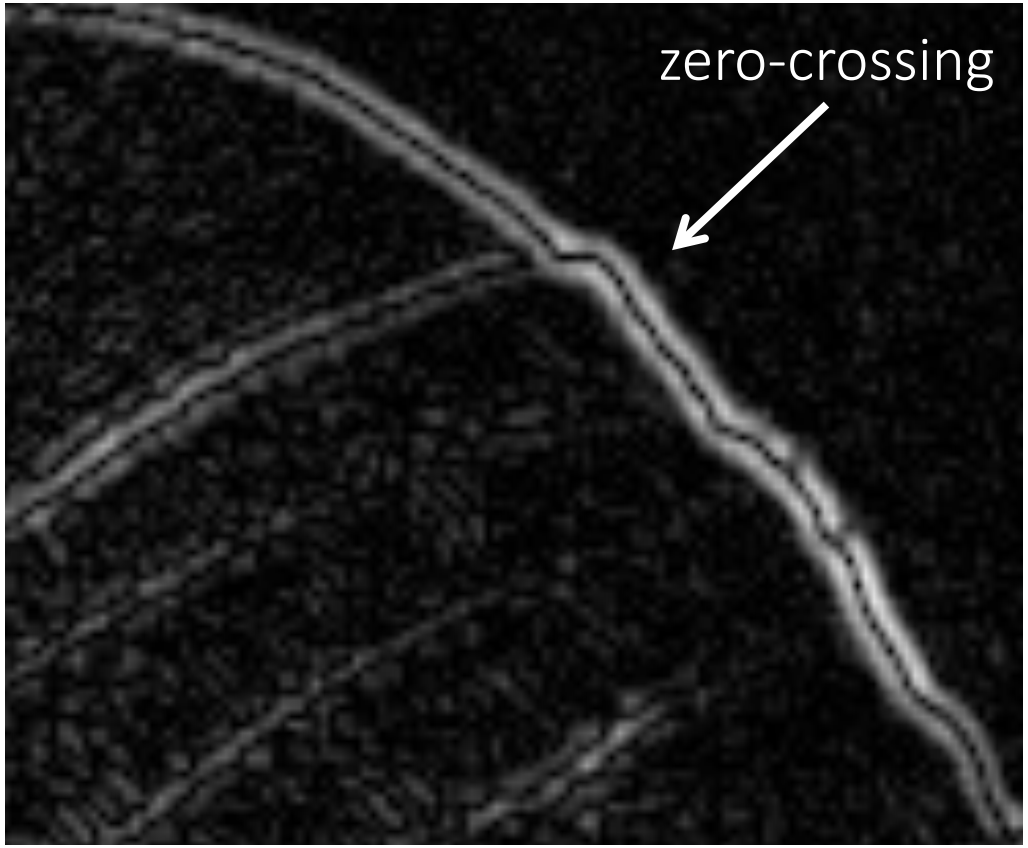
Laplace filtering

Laplacian of Gaussian vs Derivative of Gaussian

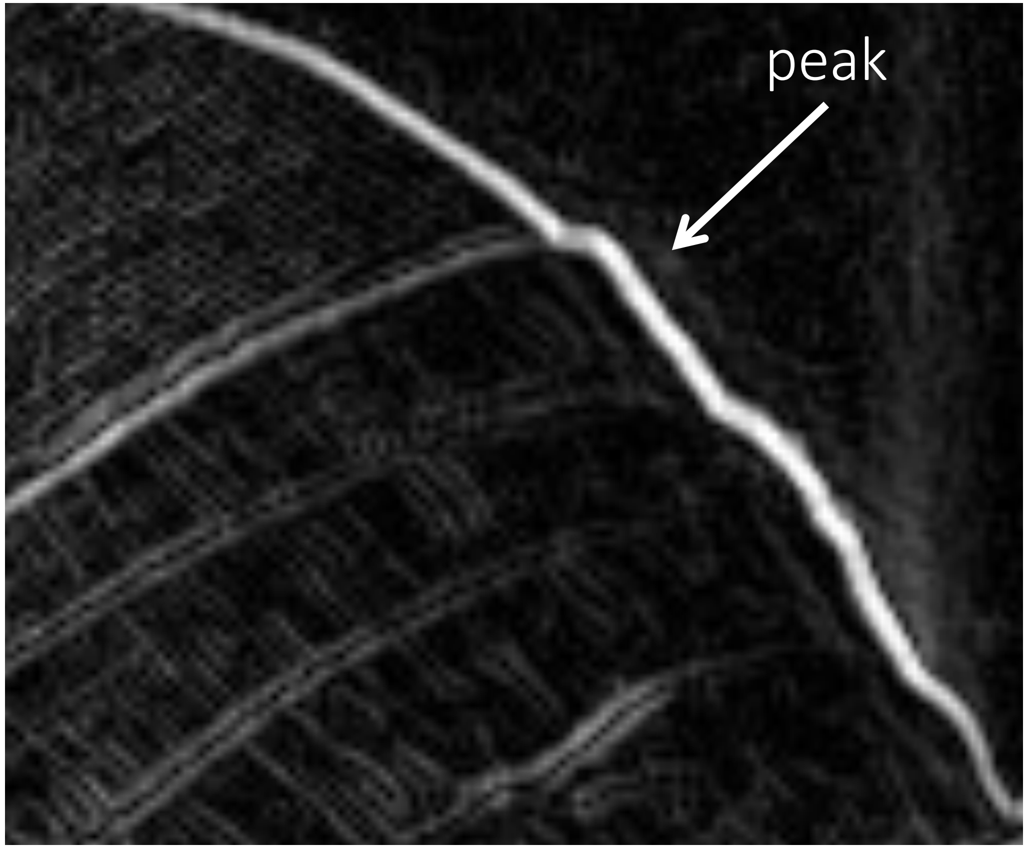
Laplacian of Gaussian filtering

Derivative of Gaussian filtering

Laplacian of Gaussian vs Derivative of Gaussian



Laplacian of Gaussian filtering



Derivative of Gaussian filtering

Zero crossings are more accurate at localizing edges (but not very convenient).

But Wait … Is Pixel Difference the Final Answer?

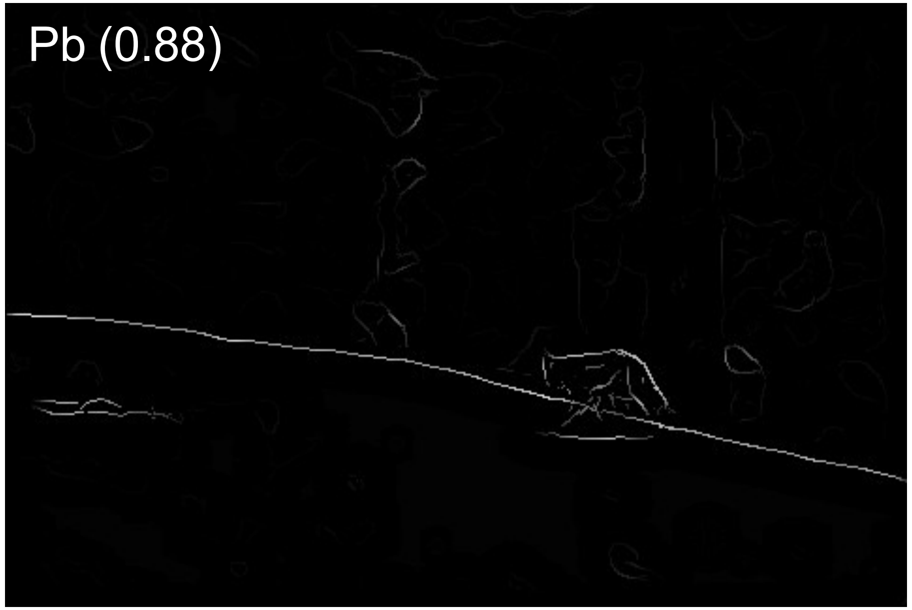
image | human segmentation | gradient magnitude

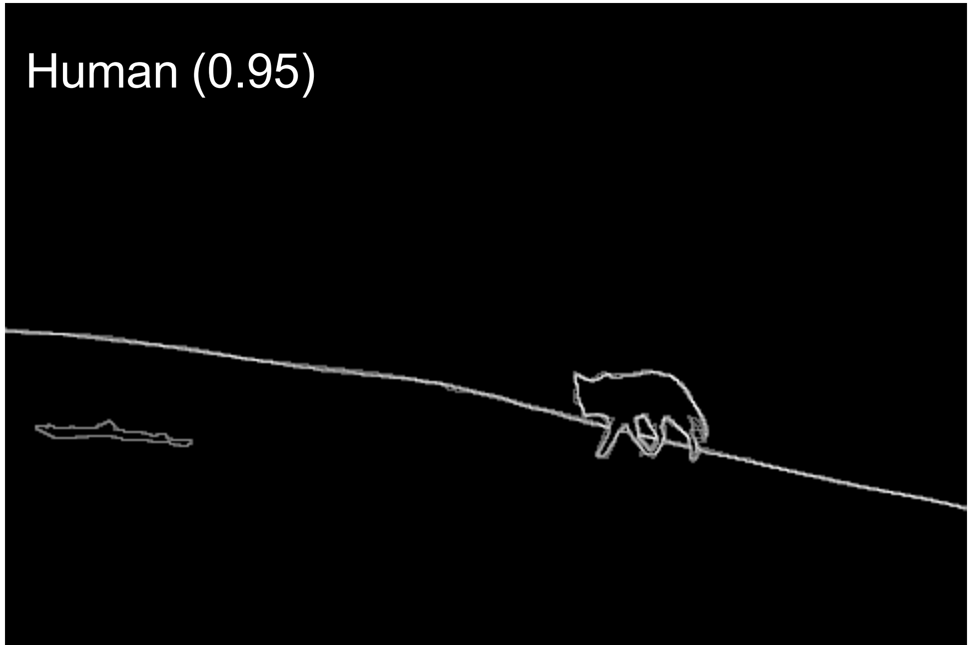
Where do humans see boundaries?

- Berkeley segmentation database:
  http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

pB slides: Hays

Score = confidence of edge.
For humans, this is averaged across multiple participants.





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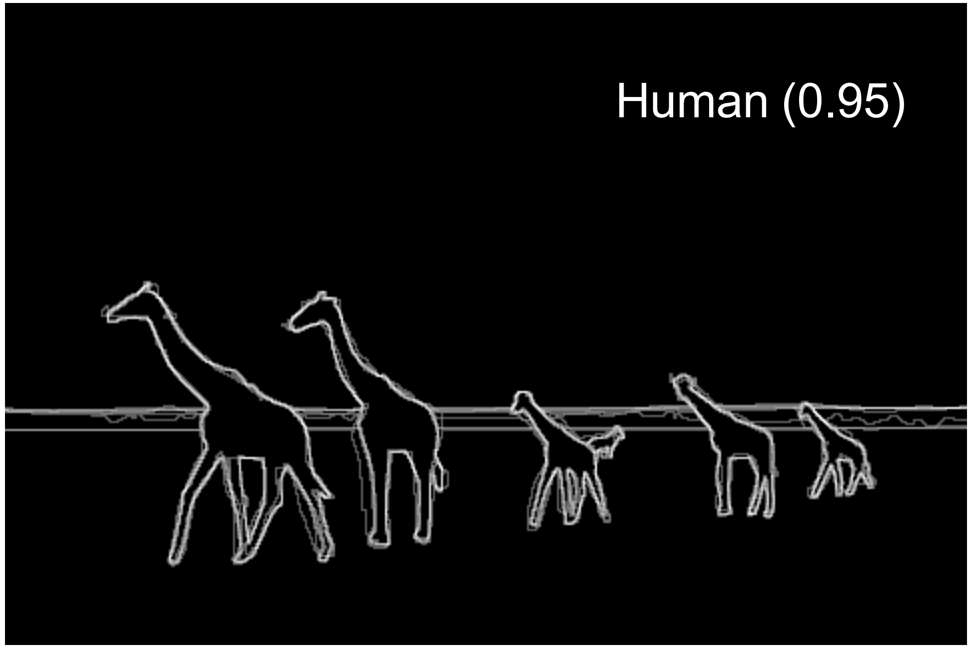


Canny Edge Detector

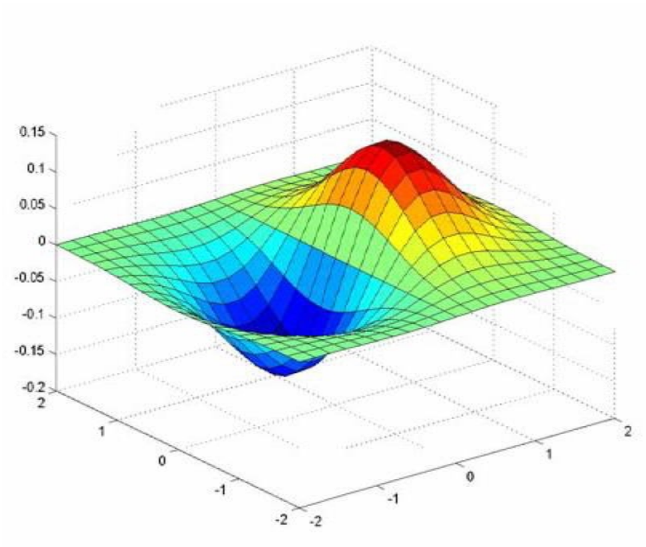
- Arguably **the most widely used** edge detector in computer vision
- Theoretical model: step-edges corrupted by additive Gaussian noise

Canny edge detector

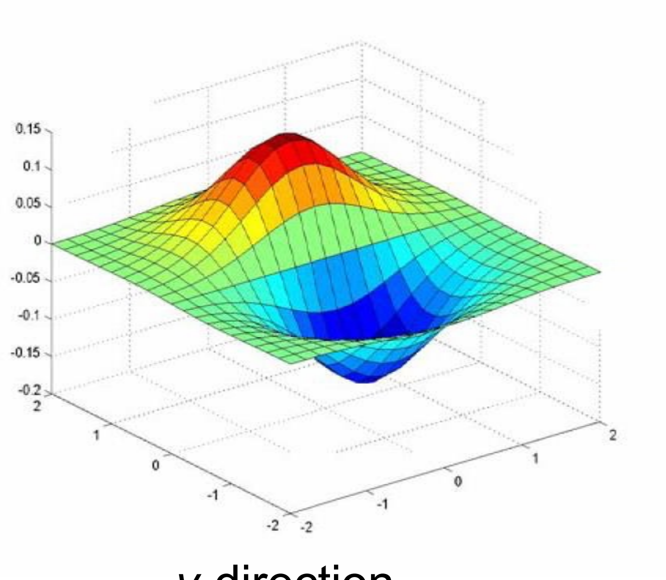
1. Filter image with x, y derivatives of Gaussian

Source: D. Lowe, L. Fei-Fei

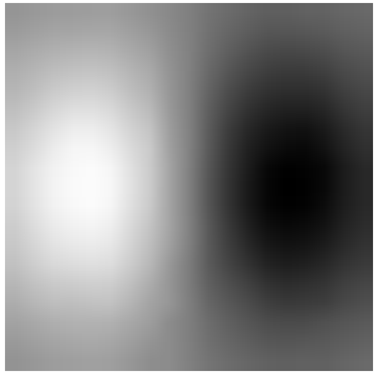
Derivative of Gaussian filter

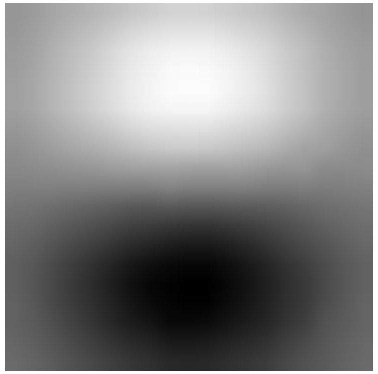


x-direction

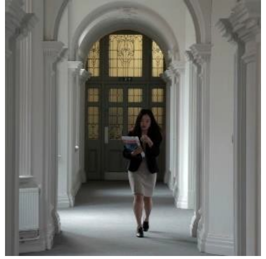


y-direction



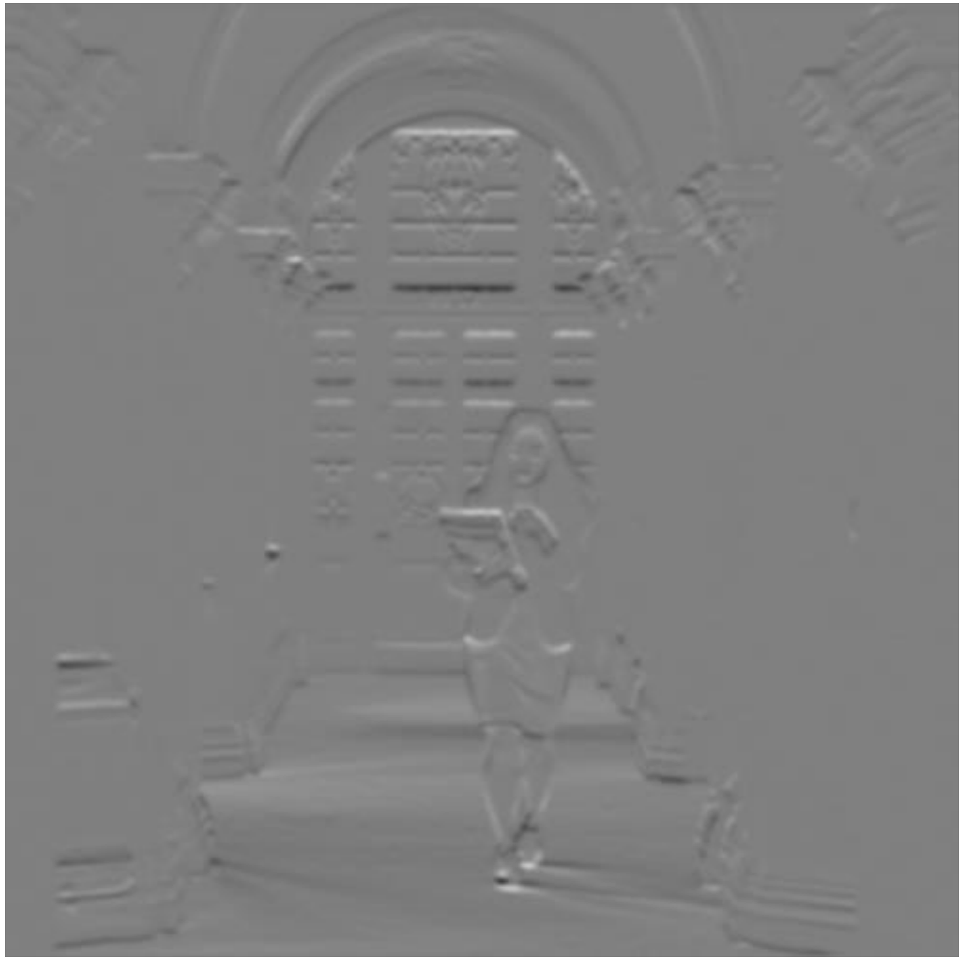


Compute Gradients



X Derivative of Gaussian

Y Derivative of Gaussian



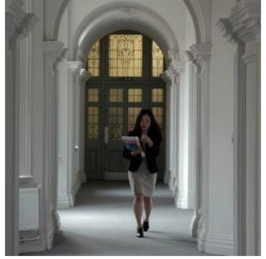
(pixel intensity x 2 + 0.5 for visualization)

Canny edge detector

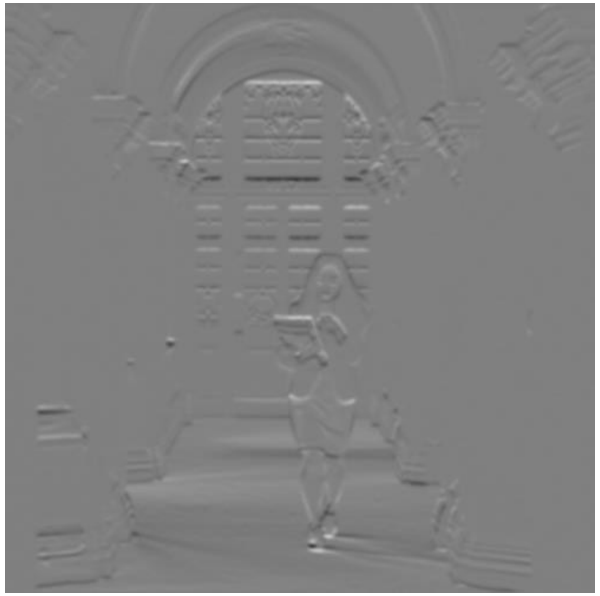
1. Filter image with x, y derivatives of Gaussian
2. Find magnitude and orientation of gradient

Source: D. Lowe, L. Fei-Fei

Compute Gradient Magnitude

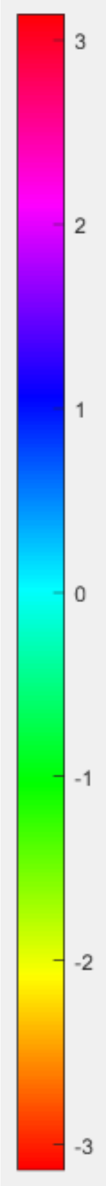


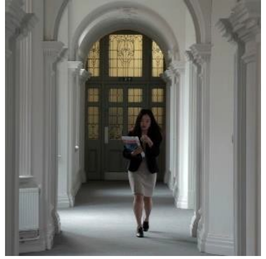
sqrt(XDerivOfGaussian .^2 + YDerivOfGaussian .^2) = gradient magnitude



Compute Gradient Orientation

- Threshold magnitude at minimum level
- Get orientation via theta = atan2(yDeriv, xDeriv)



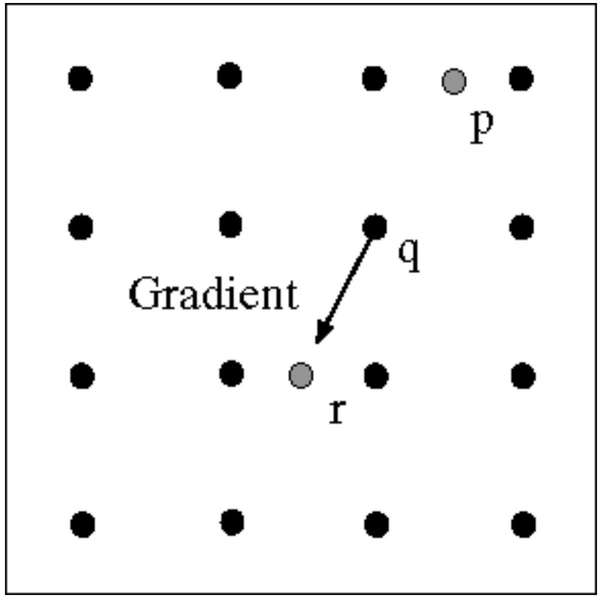


Canny edge detector

1. Filter image with x, y derivatives of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   – Thin multi-pixel wide “ridges” to single pixel width

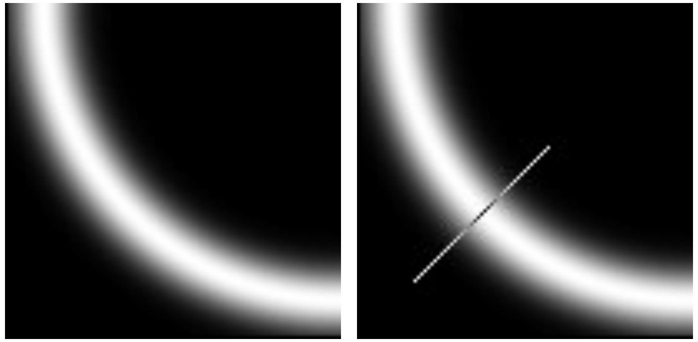
Source: D. Lowe, L. Fei-Fei

Non-maximum suppression for each orientation



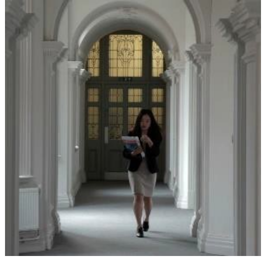
At pixel q:
We have a maximum if the value is larger than those at both p and at r.

Interpolate along gradient direction to get these values.



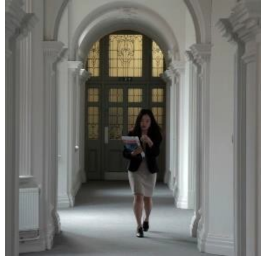
Source: D. Forsyth

Before Non-max Suppression



Gradient magnitude (x4 for visualization)

After non-max suppression



Gradient magnitude (x4 for visualization)

Canny edge detector

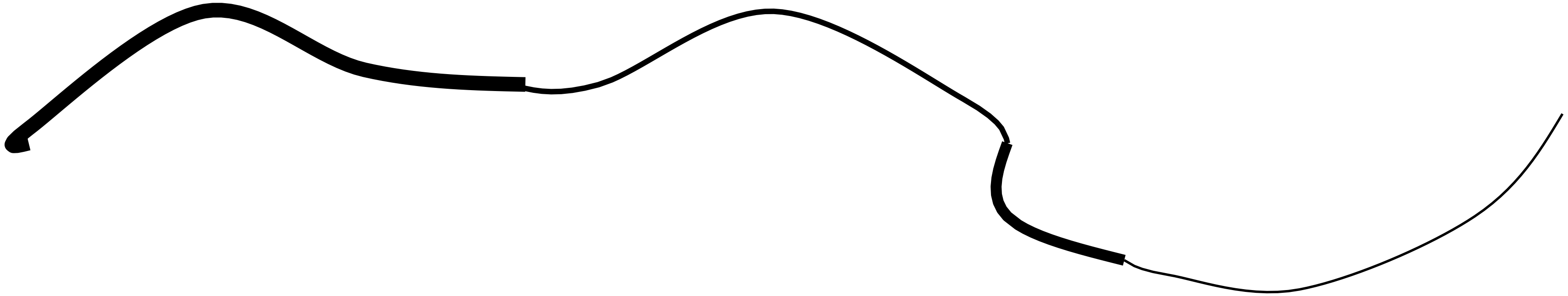
1. Filter image with x, y derivatives of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   – Thin multi-pixel wide “ridges” to single pixel width
4. ‘Hysteresis’ Thresholding

Source: D. Lowe, L. Fei-Fei

'Hysteresis' Thresholding

- Two thresholds – high and low
- Grad. mag. > high threshold? = strong edge
- Grad. mag. < low threshold? noise
- In between = weak edge

- Edge linking: 'Follow' edges starting from strong edge pixels
- Continue them into weak edges
  - Connected components



Source: S. Seitz

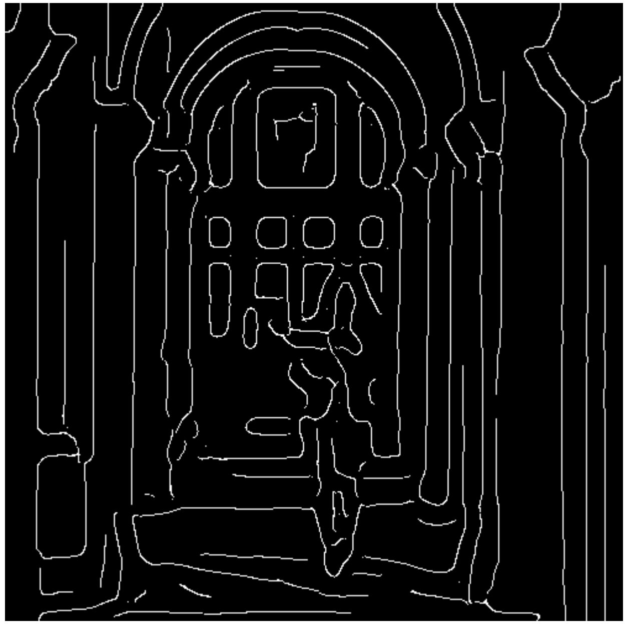
Final Canny Edges

$$\sigma = \sqrt{2}, t_{low} = 0.05, t_{high} = 0.1$$

Effect of σ (Gaussian kernel spread/size)

Original

$\sigma = \sqrt{2}$



$\sigma = 4\sqrt{2}$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

Source: S. Seitz

Canny edge detector

1. Filter image with x, y derivatives of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   - Thin multi-pixel wide “ridges” to single pixel width
4. `Hysteresis’ Thresholding:
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them
   - ‘Follow’ edges starting from strong edge pixels
     - Connected components (Szeliski 3.3.4)

Python: e.g., skimage.feature.canny()

Source: D. Lowe, L. Fei-Fei

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